# Can the Market Multiply and Divide?

# Non-Proportional Thinking in Financial Markets

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# Proportional thinking in financial markets

In financial markets, rational investors should react to news about firm value in terms of proportional price changes, i.e. ***returns***

Market value of the firm:

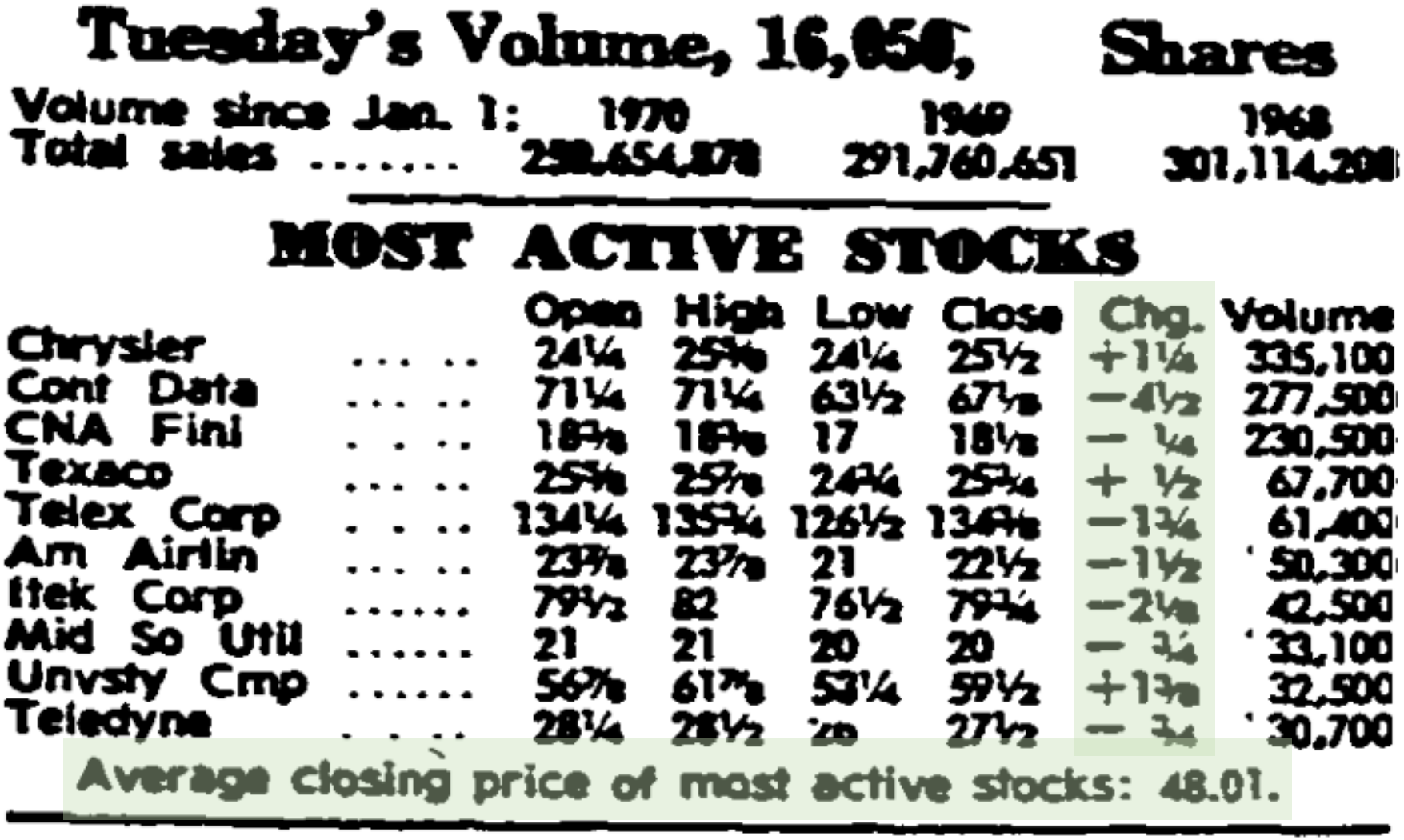
Size ≡ number of shares × share price

***Holding firm size constant***, nominal price of a financial security has no real meaning

- Price depends on the number of shares, and can easily be changed through splits or reverse splits

But changes in the value of stocks in response to news are frequently reported and discussed in dollar units…

# Wall Street Journal (1970s)



**Tuesday's Volume, 16,850, Shares**

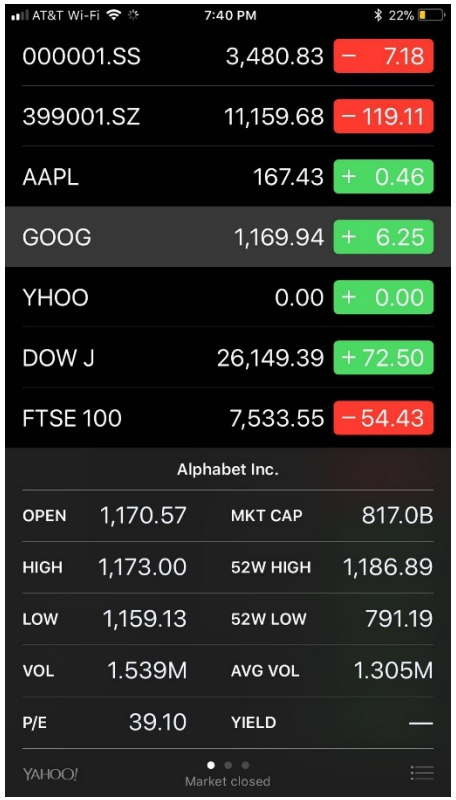
| Volume since Jan. 1: | 1970 | 1969 | 1968 |
|---|---|---|---|
| Total sales ..... | 258,654,878 | 291,760,651 | 301,114,208 |

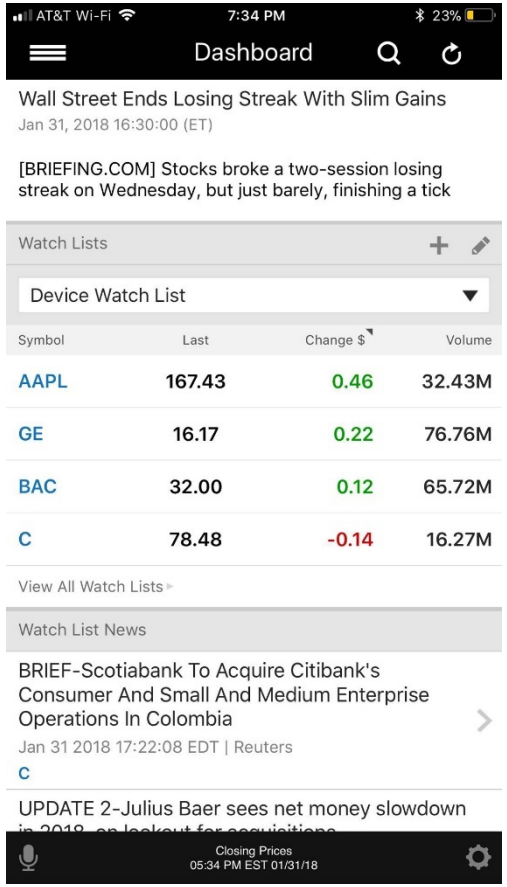
**MOST ACTIVE STOCKS**

| | Open | High | Low | Close | Chg. | Volume |
|---|---|---|---|---|---|---|
| Chrysler ... .. | 24¼ | 25⅝ | 24¼ | 25½ | +1⅛ | 335,100 |
| Cont Data ... .. | 71¼ | 71¼ | 63½ | 67⅛ | −4½ | 277,500 |
| CNA Finl . . .. | 18⅞ | 18⅞ | 17 | 18⅛ | − ¼ | 230,500 |
| Texaco ... .. | 25⅝ | 25⅞ | 24¾ | 25¾ | + ½ | 67,700 |
| Telex Corp . . .. | 134¼ | 135¾ | 126½ | 134⅝ | −1¾ | 61,400 |
| Am Airlin ... .. | 23⅞ | 23⅞ | 21 | 22½ | −1½ | 50,300 |
| Itek Corp ..... | 79½ | 82 | 76½ | 79¾ | −2⅛ | 42,500 |
| Mid So Util ..... | 21 | 21 | 20 | 20 | − ¾ | 33,100 |
| Unvsty Cmp ..... | 56⅞ | 61⅞ | 53¼ | 59½ | +1⅞ | 32,500 |
| Teledyne . . .. | 28¼ | 28½ | 26 | 27½ | − ¾ | 30,700 |

Average closing price of most active stocks: 48.01.

# Android, Apple, and Etrade apps (2010s)

# CNBC (2010s)


SQUAWK BOX
THE PRIVATE EQUITY LANDSCAPE
S&P FUT (Jun) -3.25
S&P FV 2.83
S&P CLOSE 2,047.63
Cap Bear 3X (TZA) 43.33 ▲ 0.39
iPath S&P VIX (VXX) 15.82 ▲ 0.24
Direxi
MGT Capital Invest (MGT) 2.82 ▲ 0.20
MGT Capital Invest (MGT) 2.81 ▲
CNBC
8:10A EASTERN

# Our hypothesis

Non-proportional thinking: Investors think that news should correspond to a dollar change in price rather than a percentage change

Consider two otherwise identical stocks, one trading at $20/share, another at $30/share

- Investors may think the same piece of news should correspond to a $1 increase in price for both stocks

**$1/$20 = 5%** **$1/$30 = 3.3%**

**→ Absolute magnitude of return reactions to the same news will be larger for lower priced stocks**

# Volatility predictions

Measures of a stock's volatility:

1. Total volatility: standard deviation of daily returns
2. Idiosyncratic volatility: standard deviation of returns in excess of market returns
3. Market beta: scaled covariance between the stock's return and the market return

**Stronger return reactions to news for lower-priced stocks → These stocks will have greater total volatility, idiosyncratic volatility, and market beta**

# Preview of results

A doubling in share price corresponds to 20-30% decline in volatility

- ***Not driven by size***—rather, the size-volatility relation flattens by 80% after controlling for price
- To identify a causal effect of price, we show that volatility jumps after stock splits and drops after reverse splits

Lower-priced stocks have stronger return responses to news events

Price can explain overreaction, underreaction, reversals, and drift

Not driven by tick-size limitations, volume, liquidity, or catering / changes to a speculative investor base

# Implications

**A new explanation of under and overreaction to news**

- Complements other behavioral explanations which focus on limited attention, biased beliefs about persistence

**Offers insight into the determinants of volatility and drift**

- Size doesn't matter: asset pricing facts such as "small stocks have higher volatility and market beta" are mostly driven by price
- Potential explanation for the "**leverage effect**" puzzle in which volatility is negatively related to past returns
- Reversals and predictability

**New explanation** of some **known asset pricing facts/puzzles**, e.g. Black (1976), Ohlson and Penman (1985)

# Baseline volatility regression

$$log(vol_{it}) = \beta_0 + \beta_1 log\left(price_{i,t-1}\right) + controls + \tau_t + \varepsilon_{it}$$

- Stock $i$ in year-month $t$
- $vol_{it}$: **total volatility**, **idiosyncratic volatility** or **absolute market beta**
- $controls$ can include size (linear control or 20 size categories), sales volatility, volume, bid-ask spread, institutional ownership, market-to-book, leverage, past returns, firm FE
- Standard errors double-clustered by stock and year-month

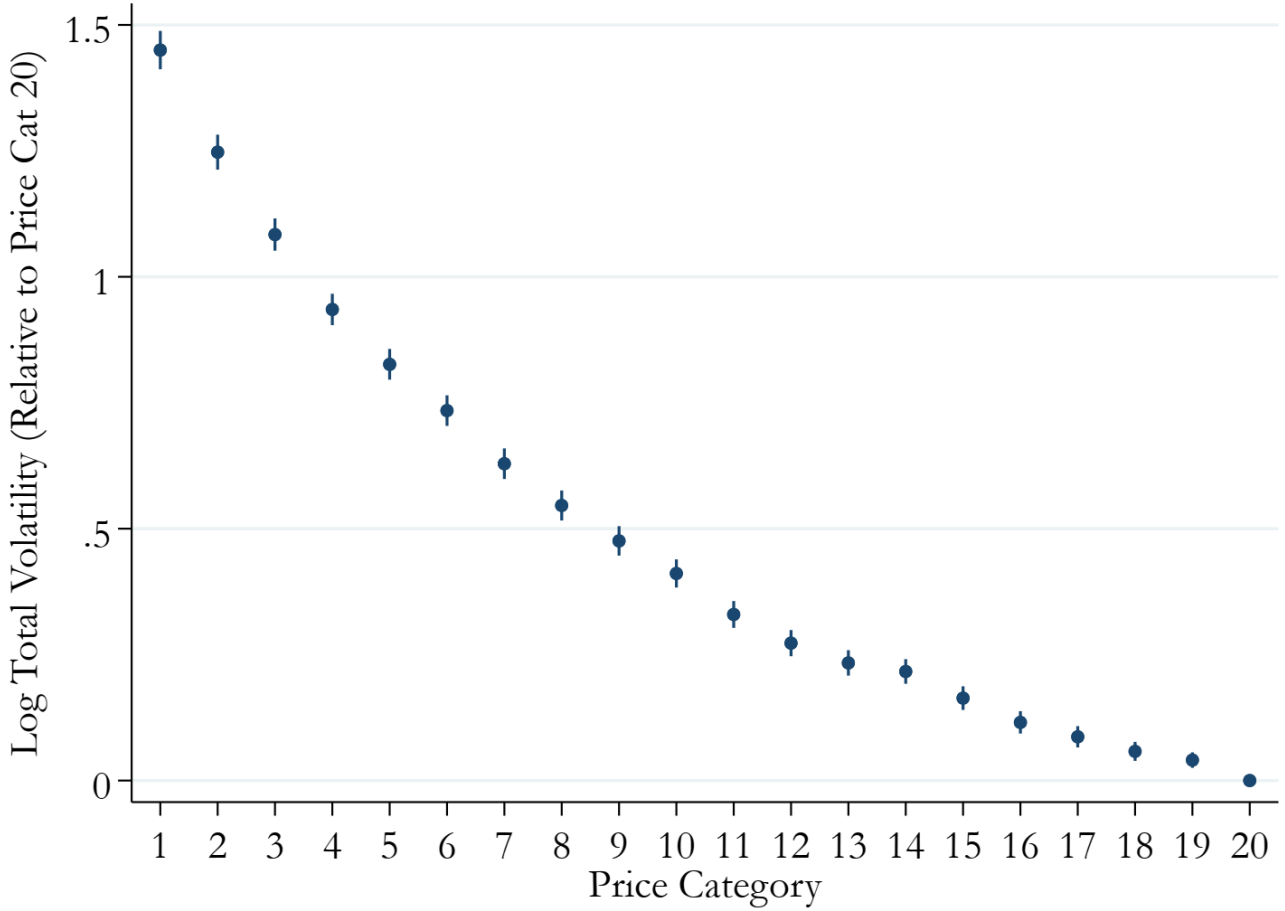
- **Non-proportional thinking predicts $\boldsymbol{\beta_1 < 0}$**

# Baseline volatility results

| | Log(Total Volatility) | | | | Log(IVol) | Log(\|Beta\|) |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Log(Lagged Price) | -0.326*** | | -0.332*** | -0.339*** | -0.346*** | -0.319*** |
| | (0.00339) | | (0.00446) | (0.00405) | (0.00399) | (0.00465) |
| Log(Lagged Size) | | -0.146*** | 0.00431 | | | |
| | | (0.00235) | (0.00311) | | | |
| Year–Month FE | Yes | Yes | Yes | Yes | Yes | Yes |
| Size Category FE | No | No | No | Yes | Yes | Yes |
| R-squared | 0.442 | 0.328 | 0.442 | 0.445 | 0.473 | 0.103 |
| Observations | 3,254,302 | 3,254,302 | 3,254,302 | 3,254,302 | 3,254,302 | 3,254,302 |

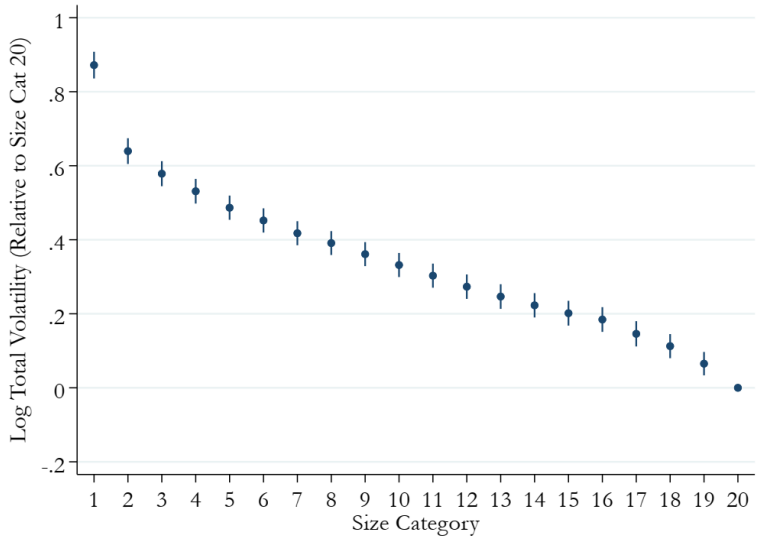
- Doubling price corresponds to a $> 30\%$
- Holds after controlling flexibly for size, yet the size-volatility relation becomes insignificant once we control for price

# Volatility-price relation

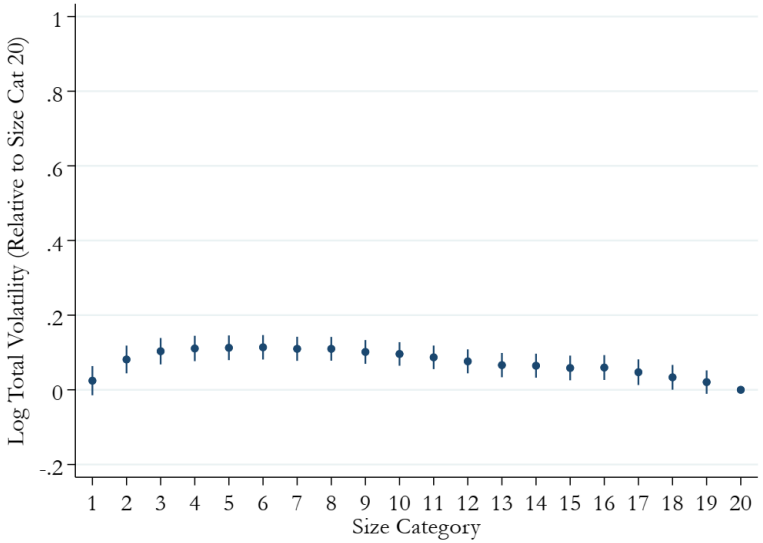



- Plots volatility against 20 price categories (omitted category 20), controlling for size categories and time
- Shows that negative relation is not driven only by low-priced stocks that are subject to tick-size distortions

# Price can explain the size-volatility relation

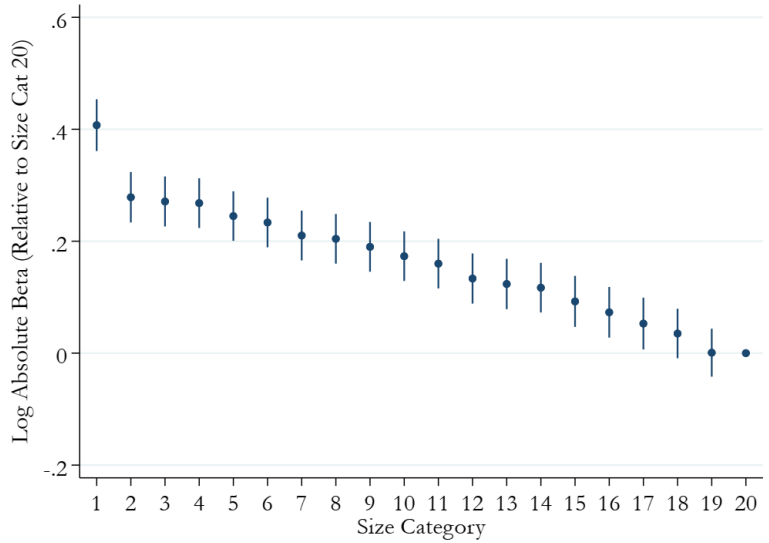



- Without controlling for $log(price_{i,t-1})$

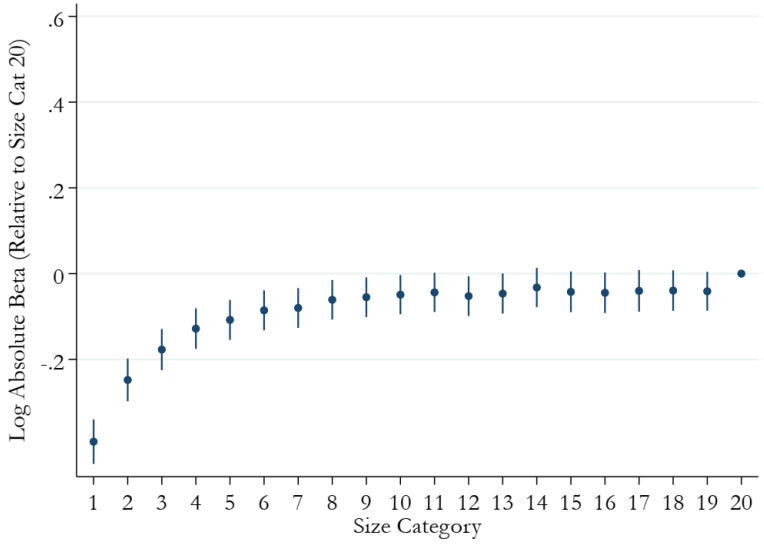



- Controlling for $log(price_{i,t-1})$
- Size-volatility relation flattens by $\sim 80\%$

# Price can explain the size-beta relation




- Without controlling for $log(price_{i,t-1})$




- Controlling for $log(price_{i,t-1})$
- Size-beta relation flattens by $\sim 80\%$

# The leverage effect puzzle

**Puzzle: Past returns are strongly negatively related to volatility**

**Several potential explanations, including…**

- *Leverage Effect*: Holding the debt level constant, if asset value falls, then the equity becomes more levered and consequently more risky
  - Problem: same pattern holds for firms with zero leverage

**Non-proportional thinking**

- Negative returns imply a decline in price
- Reacting to news in nominal units leads to higher volatility for lower-priced stocks

# The leverage effect puzzle

| | Log(Total Volatility) | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Log(Lagged Price) | -0.339*** | | -0.332*** | -0.331*** |
| | (0.00412) | | (0.00465) | (0.00466) |
| Log(Past 12-Month Return) | | -0.240*** | -0.0309*** | |
| | | (0.0101) | (0.00902) | |
| Year–Month FE | Yes | Yes | Yes | Yes |
| Size Category FE | Yes | Yes | Yes | Yes |
| Past 12 Monthly Returns | No | No | No | Yes |
| R-squared | 0.458 | 0.346 | 0.458 | 0.459 |
| Observations | 2,966,196 | 2,966,196 | 2,966,196 | 2,966,196 |

# Heterogeneity by size

| | Log(Total Volatility) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) Size1 | (2) Size2 | (3) Size3 | (4) Size4 | (5) Size5 | (6) Size6 | (7) Size7 | (8) Size8 | (9) Size9 | (10) Size10 |
| Log(Lagged Price) | -0.363*** | -0.386*** | -0.370*** | -0.364*** | -0.346*** | -0.325*** | -0.310*** | -0.298*** | -0.281*** | -0.270*** |
| | (0.00562) | (0.00659) | (0.00746) | (0.00771) | (0.00800) | (0.00816) | (0.00870) | (0.00870) | (0.00949) | (0.00996) |
| Year–Month FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| R-squared | 0.393 | 0.368 | 0.363 | 0.362 | 0.354 | 0.349 | 0.347 | 0.351 | 0.343 | 0.350 |
| Observations | 1,111,769 | 333,979 | 226,006 | 173,581 | 146,796 | 128,577 | 117,145 | 107,699 | 99,812 | 92,354 |

| | Log(Total Volatility) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) Size11 | (2) Size12 | (3) Size13 | (4) Size14 | (5) Size15 | (6) Size16 | (7) Size17 | (8) Size18 | (9) Size19 | (10) Size20 |
| Log(Lagged Price) | -0.248*** | -0.227*** | -0.207*** | -0.201*** | -0.184*** | -0.168*** | -0.166*** | -0.124*** | -0.123*** | -0.139*** |
| | (0.00969) | (0.0103) | (0.0118) | (0.0123) | (0.0135) | (0.0139) | (0.0179) | (0.0165) | (0.0206) | (0.0215) |
| Year–Month FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| R-squared | 0.355 | 0.359 | 0.351 | 0.357 | 0.359 | 0.388 | 0.403 | 0.412 | 0.450 | 0.526 |
| Observations | 85,267 | 81,440 | 78,053 | 75,213 | 72,894 | 70,567 | 67,662 | 65,314 | 62,743 | 57,431 |

Magnitude of volatility-price relation declines with size, but still strong for the largest 5% of stocks

# Heterogeneity by decade

| | Log(Total Volatility) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) 1920s | (2) 1930s | (3) 1940s | (4) 1950s | (5) 1960s | (6) 1970s | (7) 1980s | (8) 1990s | (9) 2000s | (10) 2010s |
| Log(Lagged Price) | -0.227*** | -0.275*** | -0.350*** | -0.191*** | -0.324*** | -0.462*** | -0.317*** | -0.369*** | -0.353*** | -0.251*** |
| | (0.0140) | (0.0108) | (0.00989) | (0.0147) | (0.0126) | (0.00862) | (0.00646) | (0.00768) | (0.00995) | (0.00554) |
| Year–Month FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Size Category FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| R-squared | 0.560 | 0.652 | 0.609 | 0.289 | 0.396 | 0.353 | 0.315 | 0.440 | 0.449 | 0.356 |
| Observations | 22,843 | 82,661 | 97,620 | 118,906 | 209,217 | 452,864 | 624,639 | 751,051 | 586,932 | 307,569 |

Some variation over time, but the relation is not only driven by the early sample period

# Splits as an event study

Potential concern: poor performance leads to low price and high vol

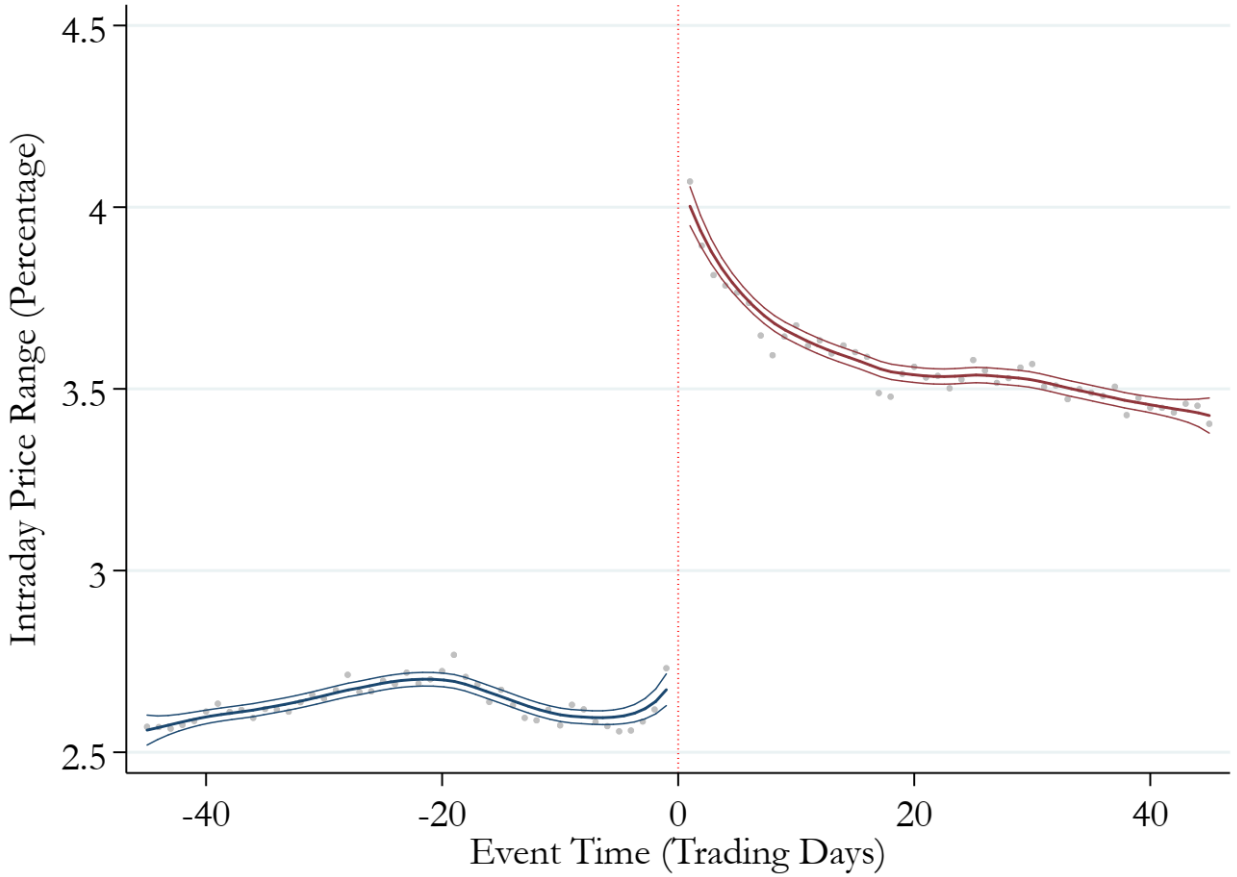
To identify a causal effect of price, we conduct a regression discontinuity around stock splits

- Following a standard 2-for-1 stock split, the price falls by half

Splits are not random (they tend to follow good performance), but splits are pre-scheduled and fundamentals are unlikely to change exactly on the split execution date

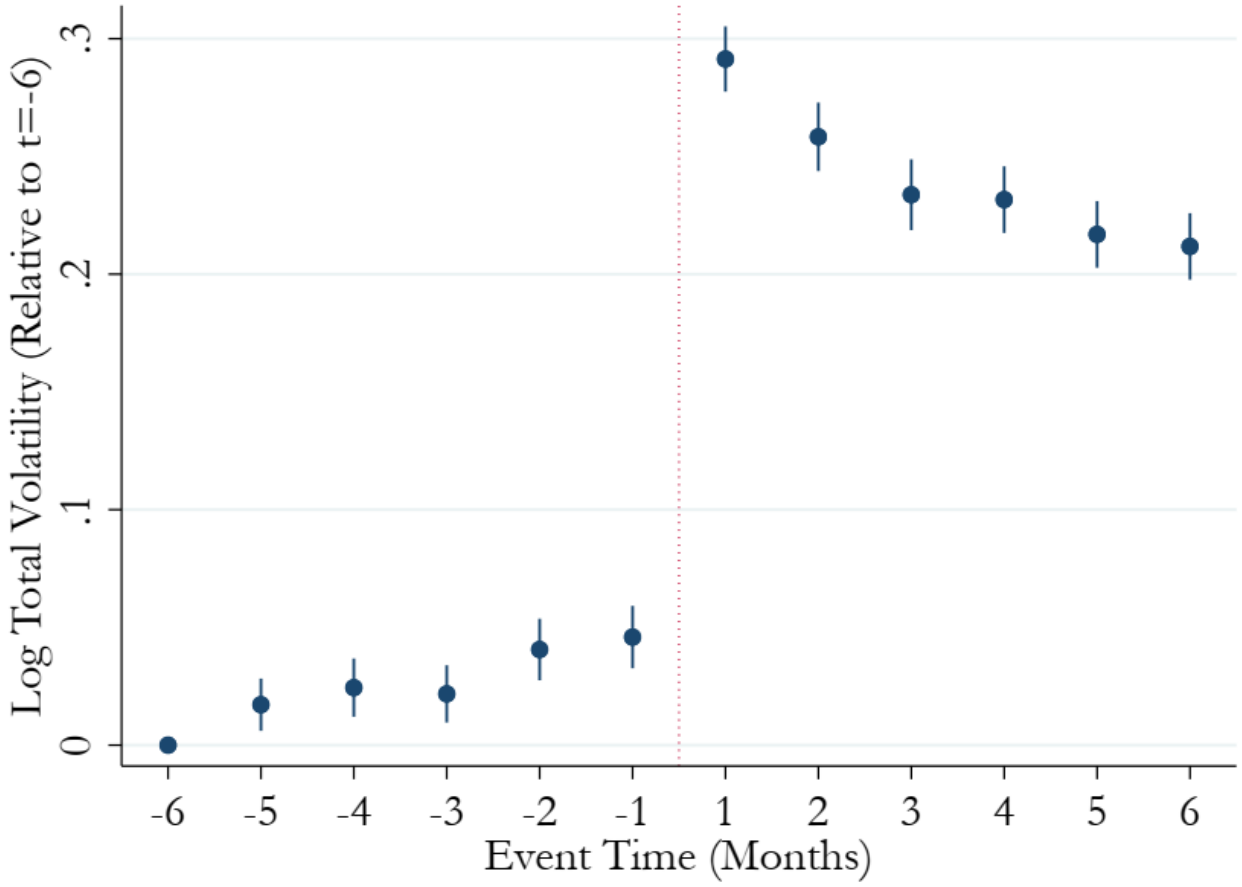
Expect the opposite patterns for reverse splits

# Regression discontinuity around stock splits



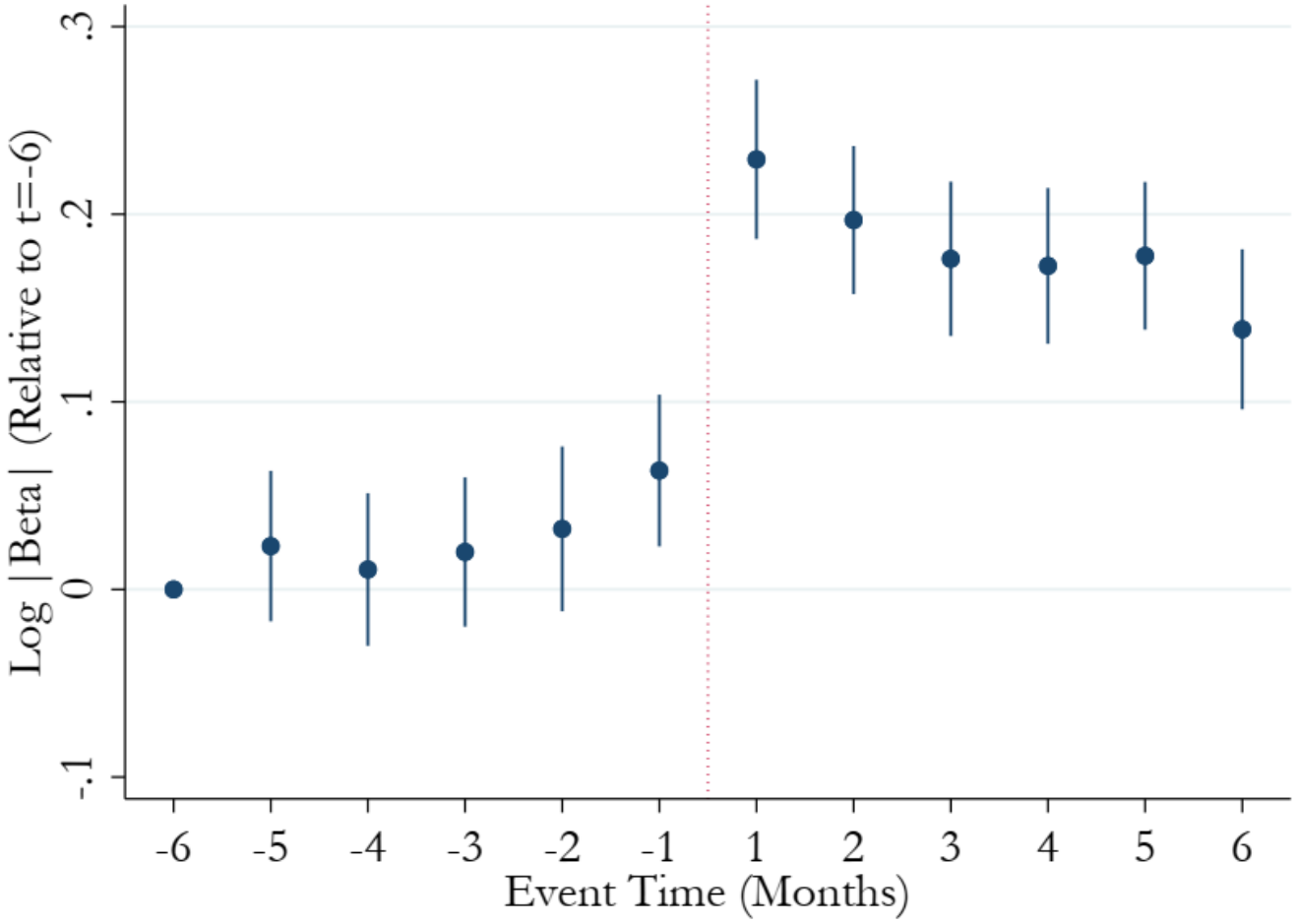

- Proxy for daily volatility using intraday price range percentage
- $> 30\%$ persistent increase in intraday price range after splits

# Splits (total volatility)

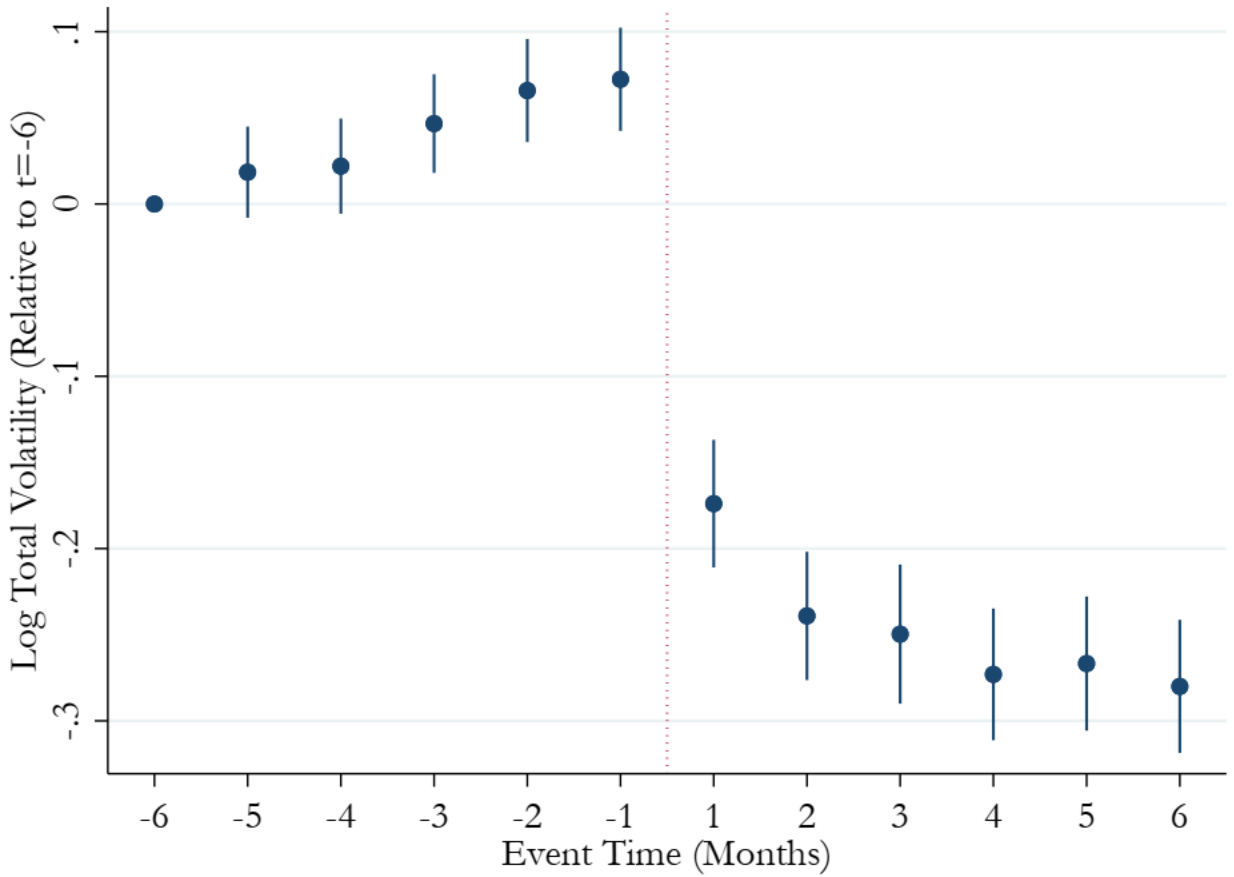



Regression coefficients on event months ($t = -6$ is omitted), controlling for stock and calendar year-month FE

# Splits (market beta)



# Reverse splits




Following reverse splits: prices↑, volatility↓

# Remaining alternative explanations

Low prices may attract speculative investors who push up volatility

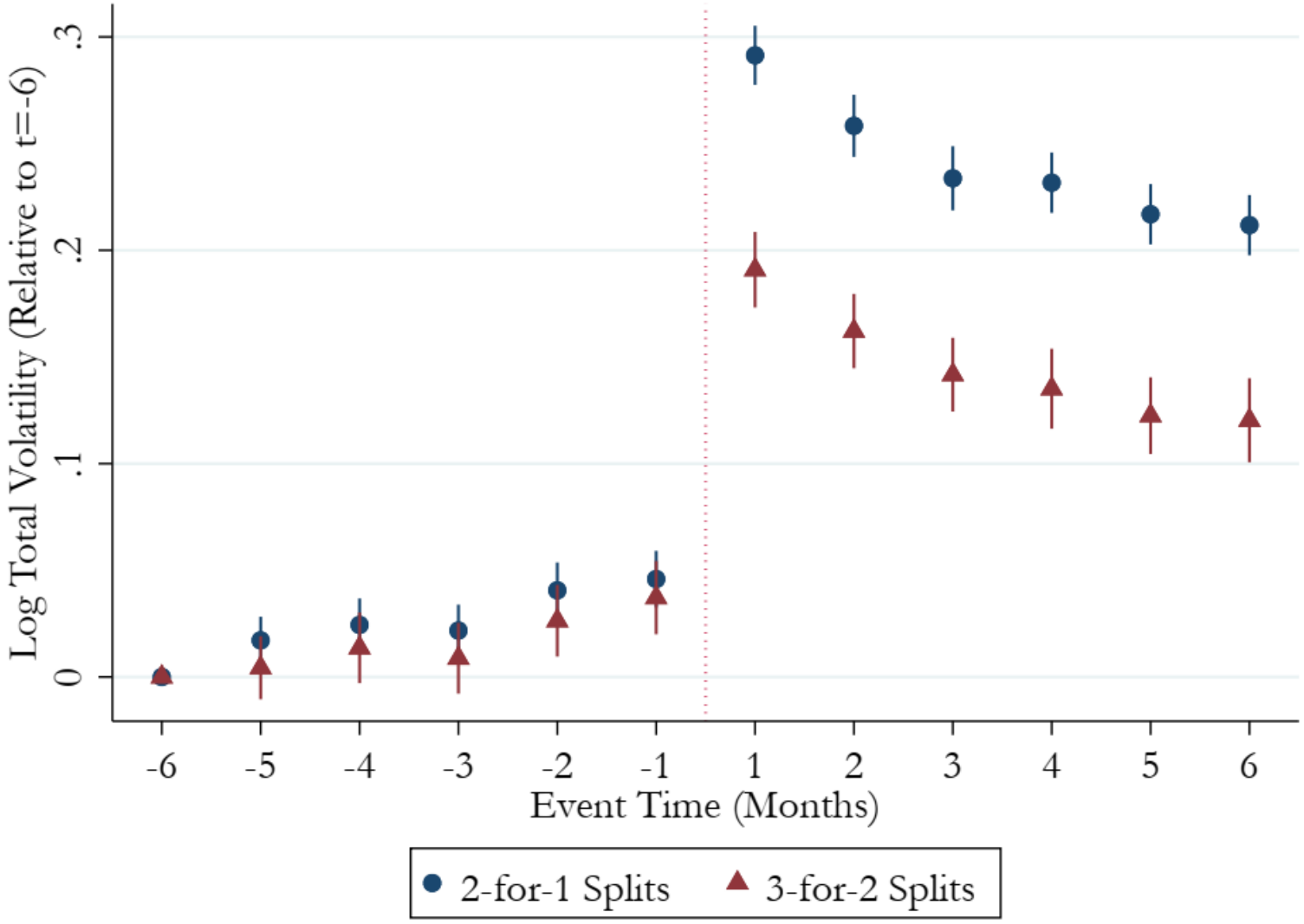
- Unlikely that investor base changes in a single day after split
- Not obvious that speculative investors would overreact to news, leading to higher betas and subsequent reversals

Firms may announce splits when they expect changes in firm strategy/performance, which could affect volatility

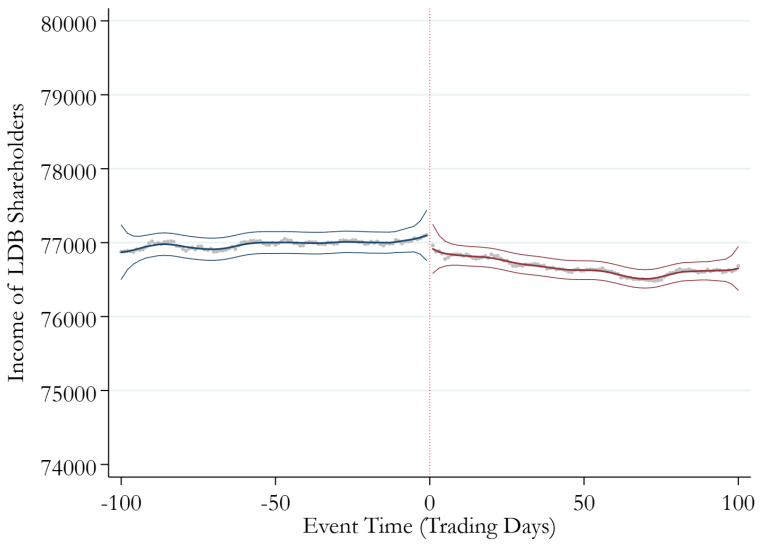
- However, splits are usually announced one month ahead

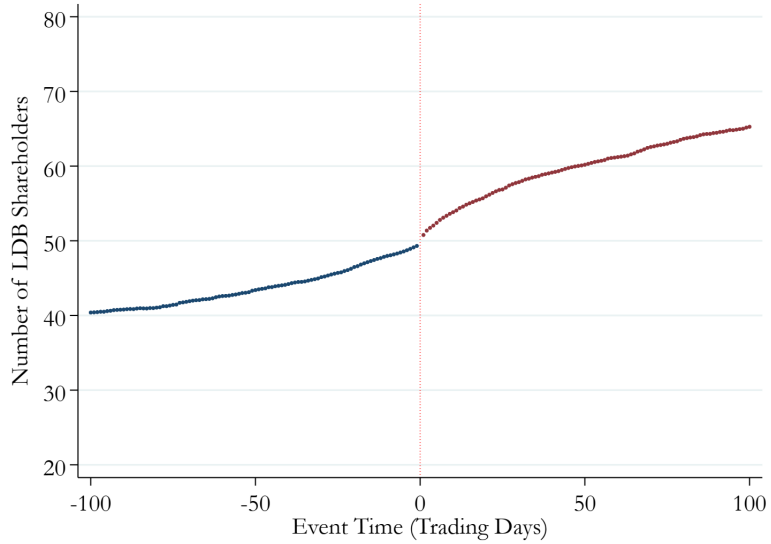
We can also examine these stories in more detail…

# Effect size can be ranked by the type of split



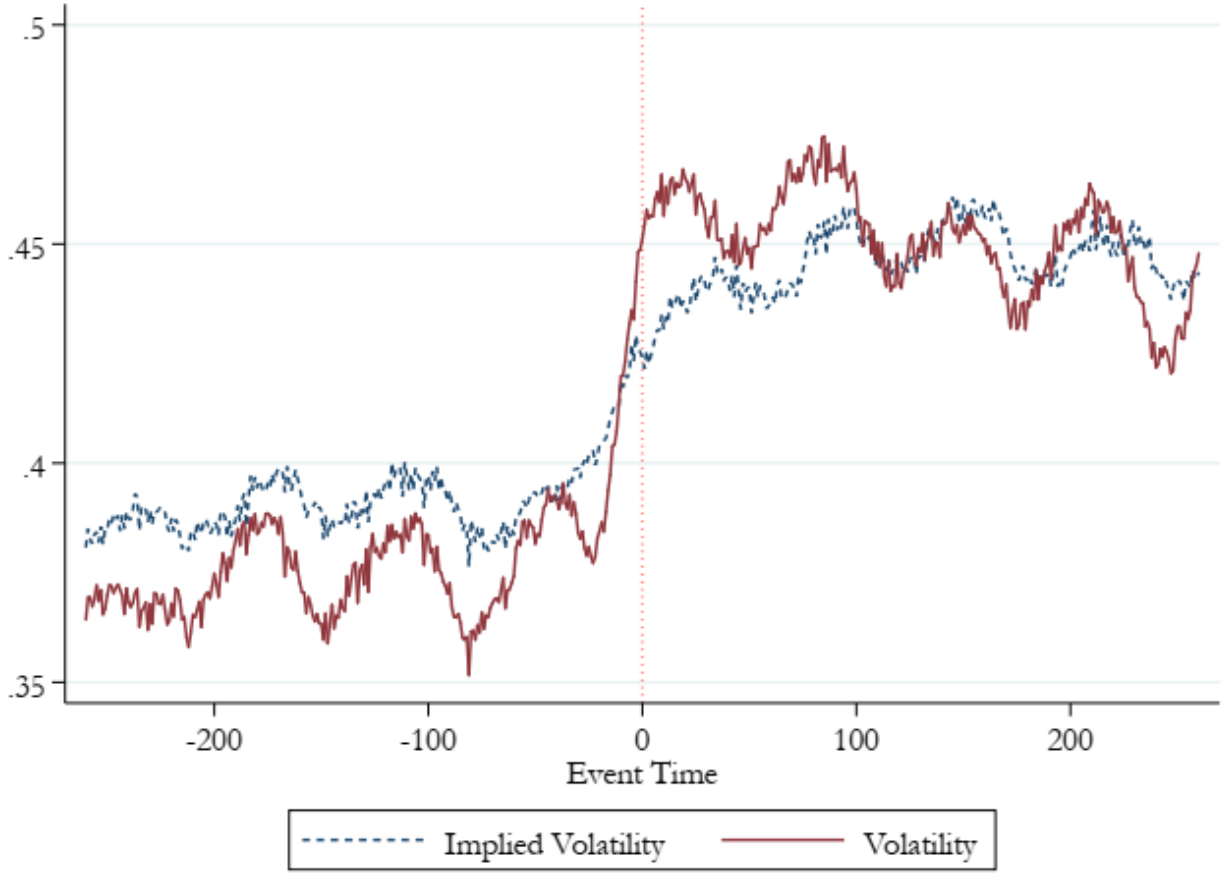
# Small changes in retail investor base



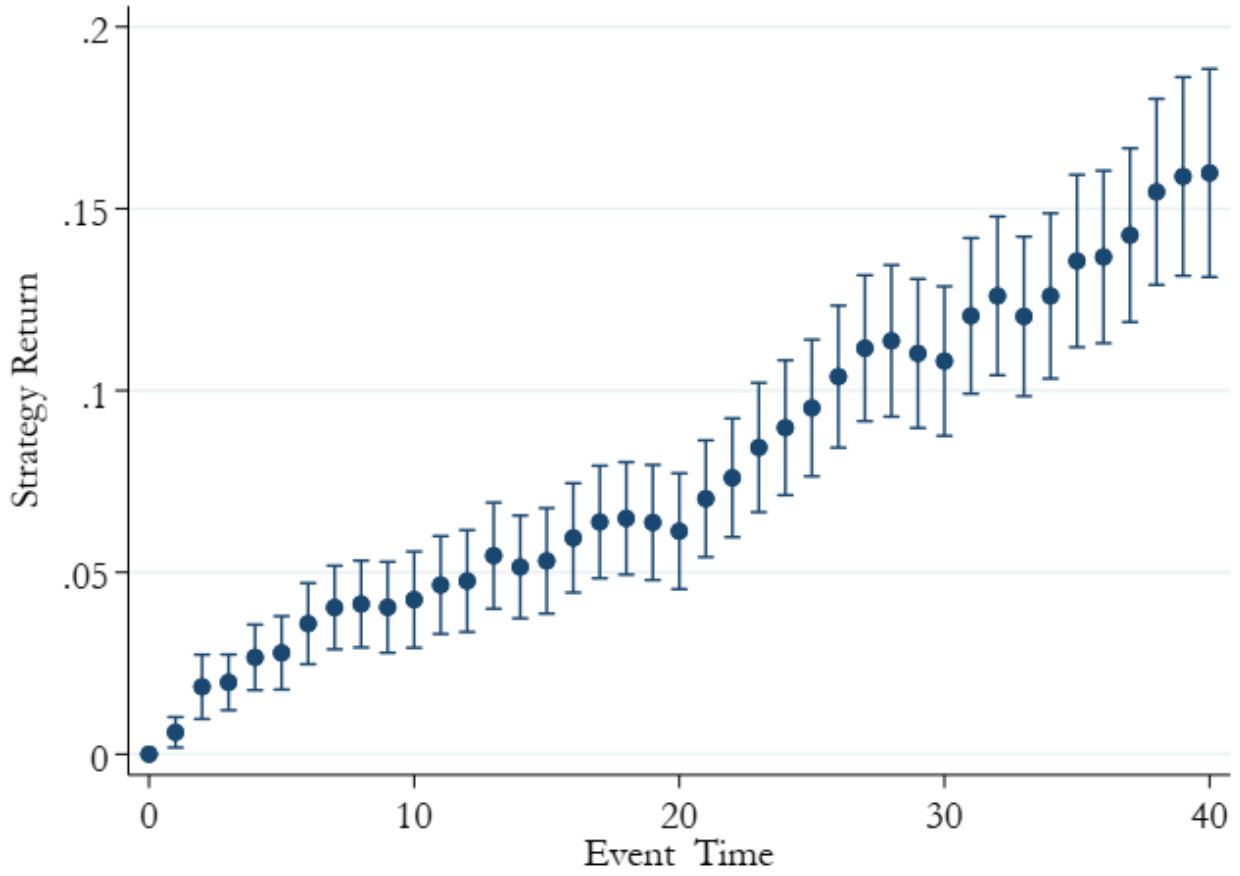

The 20-30% jump in volatility is ***too large*** to be explained by the small change in retail trading

# Option implied vol and actual vol




Option traders under-estimate the increase in volatility after splits

# Trading strategy: buy straddles on split date




Average 15 percent return within 40 days after split (does not account for transaction costs)

# How do managers think about splits?

**Are CEOs aware that splits lead to a 20-30% increase in volatility?**

- Self-interested CEOs should love splits, because the jump in volatility would increase the value of executive stock options
- Other CEOs may avoid splits to keep volatility and beta low

**Commonly-discussed reason for splits: Attract a broader investor base and increase liquidity**

- In reality, a greater number of different investors trade the stock after a split, but volume turnover falls because some investors don't realize they should double the number of shares traded

# Responsiveness to news

**So far, we've focused on volatility**

**Non-proportional thinking also predicts**

- Stronger return reaction to news for lower-priced stocks

**Caveat:** We don't know whether stocks overreact or underreact in an absolute sense

- Investors could underreact for other reasons (e.g. limited attention), but underreact to a *lesser extent* for lower priced stocks due to non-proportional thinking

# News identified from textual analysis

Textual analysis of firm-specific news, **S&P 500 firms in 2000s**

- Categorized news: value-relevant events, e.g. M&A, products

| | Log(\|CAR\|) | | | |
|---|---|---|---|---|
| | Categorized News | | Other News | |
| | (1) | (2) | (3) | (4) |
| Log(Lagged Price) | -0.299*** | -0.217*** | -0.279*** | -0.208*** |
| | (0.0161) | (0.0161) | (0.0156) | (0.0169) |
| Year–Month FE | Yes | Yes | Yes | Yes |
| Size Category FE | No | Yes | No | Yes |
| R-squared | 0.105 | 0.115 | 0.106 | 0.111 |
| Observations | 377,454 | 377,454 | 375,123 | 375,123 |

# Reactions to earnings news

Quarterly earnings are an important type of firm-specific news

| | Log(\|CAR\|) | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Log(Lagged Price) | -0.223*** | -0.232*** | -0.271*** | -0.210*** |
| | (0.00747) | (0.0102) | (0.00791) | (0.00944) |
| Year–Month FE | Yes | Yes | Yes | Yes |
| Size Category FE | No | Yes | No | Yes |
| Analyst Count FE | No | No | Yes | Yes |
| R-squared | 0.067 | 0.071 | 0.072 | 0.082 |
| Observations | 339,736 | 339,736 | 339,736 | 339,736 |

A doubling in share price corresponds to a >20% reduction in the absolute return response in a 3-day window around earnings announcements

# Reversals

**Non-proportional thinking predicts**

- Potential overreaction to news for low-priced stocks, followed by *eventual reversal*

**Classic evidence of long run reversals:** De Bondt and Thaler (1985)

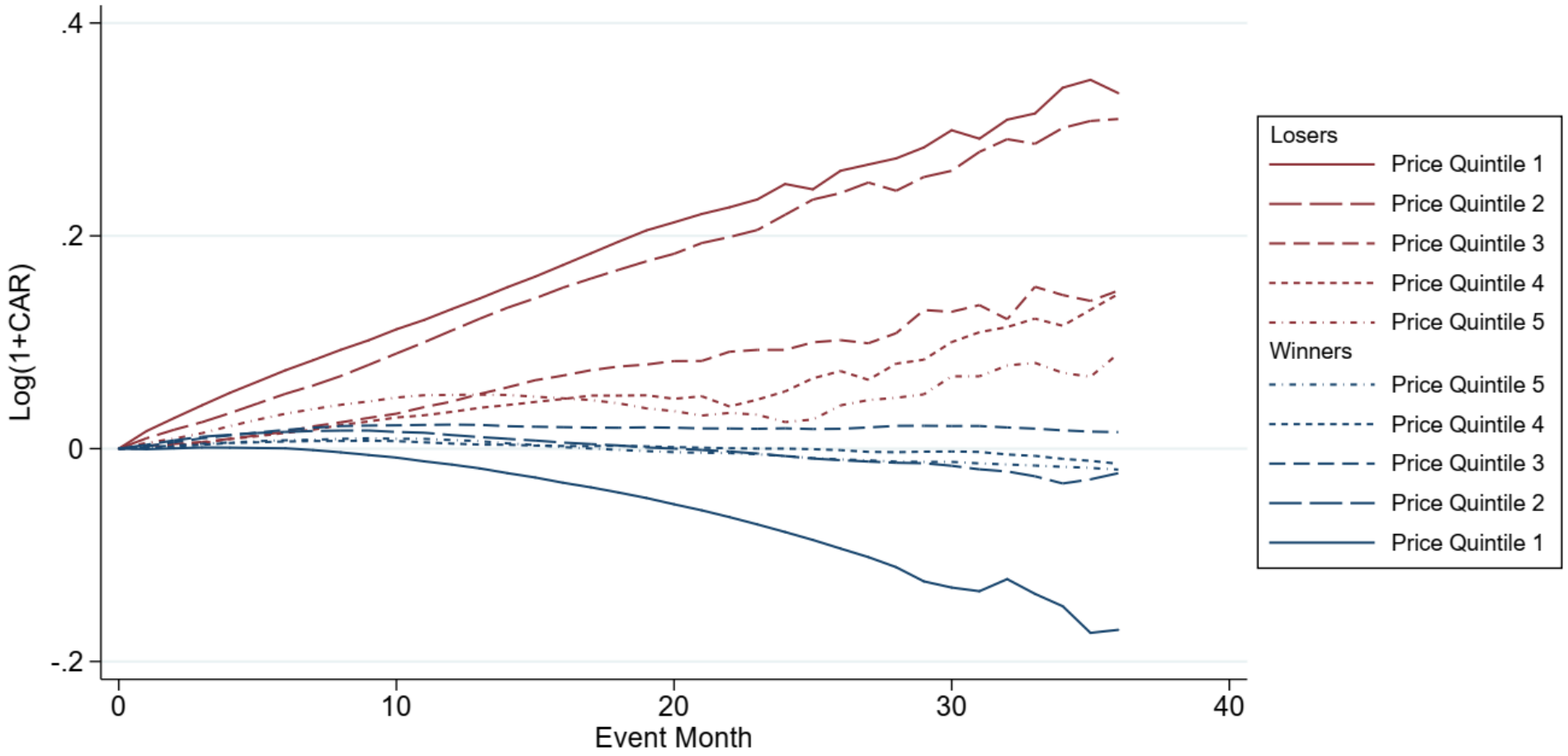
- Past winners underperform, past losers outperform

We can replicate De Bondt and Thaler (1985), and find

- The reversal is driven by low-priced stocks
- The magnitude of the reversal can be better sorted by price than size

**Short run reversal** (Jegadeesh 1990) is similarly better sorted by price than by size

# Long run reversals: by price




- Overreaction to news for low-priced stocks and eventual reversal
- Underreaction to news for high-priced stocks and eventual drift

# Conclusion

Non-proportional thinking: Investors think that news should correspond to a dollar change in price rather than a percentage change in price

- Stronger return reactions to news for lower-priced stocks

Economic magnitudes are large

- NPT can explain a significant portion of the “leverage effect” puzzle as well as the volatility-size and beta-size relations in the data

Offers insight into the determinants of over- and underreaction, volatility, drift, and reversals

# Practical takeaways

**Many trading strategies bet on drift (momentum) or reversals**

- Strategies are known to perform better for small-caps
- Downside of small caps: High transaction costs and price impact
- Our findings suggest that it would be more cost-effective to target low-priced stocks for reversals and high-priced stocks for momentum

**Investors should be cautious of news reported in the wrong units**

- Dollars vs. percents
- Real dollars vs. nominal dollars
- Index points vs. percents
- S&P 500 index does not include dividend payouts
- Dow is a price-weighted index
- Earnings per share surprise of $0.05 per share is typically bigger news for a $20 stock than $30 dollar stock

# Relation to the proportional-thinking bias

*Consumers are often willing to drive to another store to save $10 off a $20 calculator but not to save $10 off a $100 jacket*

Our results suggest people think partly in dollars and partly in percents, leading to mistakes in settings in which:

- They should think entirely in dollars (consumer choice)
- They should think entirely in percents (financial markets)

# Reactions to earnings news

Advantage: can control for the actual earnings news

| | Log(1+CAR) | | | |
|---|---|---|---|---|
| | All SUE Deciles | | Extreme SUE Deciles | |
| | (1) | (2) | (3) | (4) |
| SUE Decile Rank | 0.00994*** | 0.00995*** | | |
| | (0.000532) | (0.000533) | | |
| SUE Decile Rank × Lagged Price Quintile 2 | -0.000366 | -0.000385 | | |
| | (0.000512) | (0.000514) | | |
| SUE Decile Rank × Lagged Price Quintile 3 | -0.00180*** | -0.00180*** | | |
| | (0.000550) | (0.000551) | | |
| SUE Decile Rank × Lagged Price Quintile 4 | -0.00360*** | -0.00359*** | | |
| | (0.000538) | (0.000539) | | |
| SUE Decile Rank × Lagged Price Quintile 5 | -0.00443*** | -0.00442*** | | |
| | (0.000548) | (0.000549) | | |

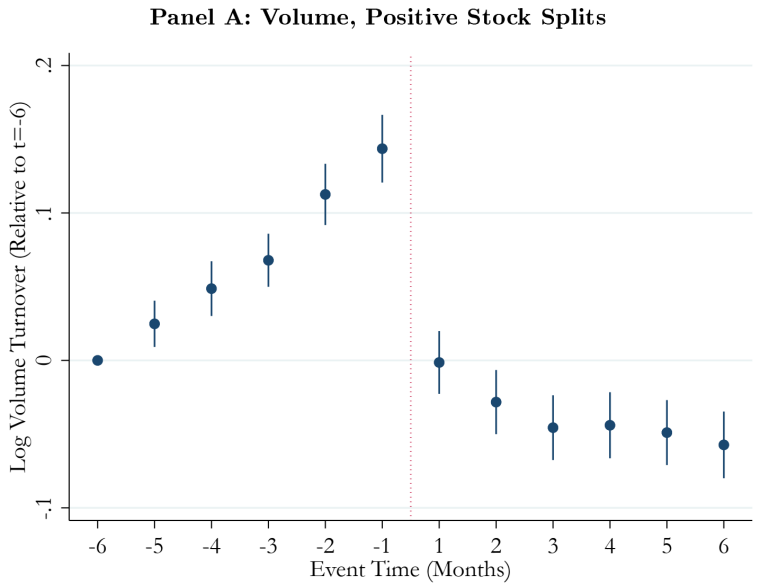
Controlling for actual earnings news, return response is stronger for lower priced stocks

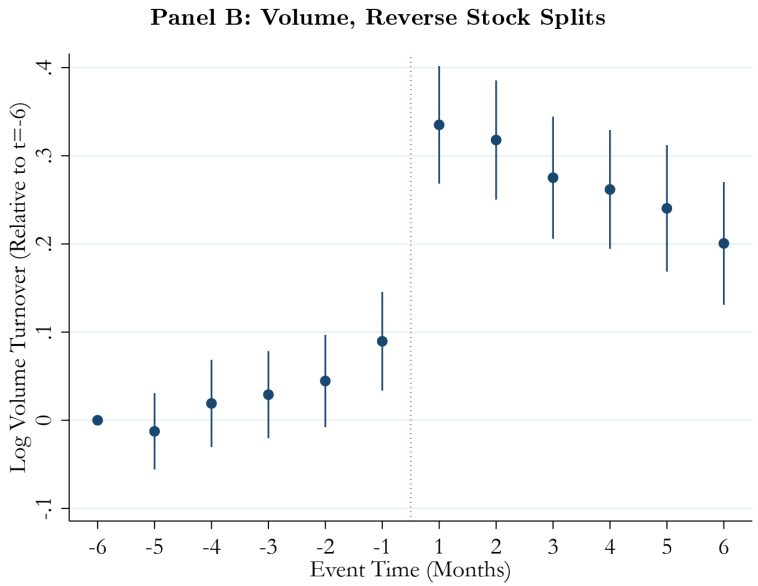
# Reversals: sorting by **price** vs **size**

| | Log(1 + 36 Month CAR) | | | Log(1 + 1 Month CAR) | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Log(1 + Prev 36 Month CAR) | -0.154*** | -0.186*** | -0.158*** | | | |
| | (0.00598) | (0.00982) | (0.00984) | | | |
| Log(1 + Prev 36 Month CAR) × Log($Price_{t-37}$) | 0.0311*** | | 0.0314*** | | | |
| | (0.00183) | | (0.00216) | | | |
| Log(1 + Prev 36 Month CAR) × Log($Size_{t-37}$) | | 0.0112*** | 0.000188 | | | |
| | | (0.00106) | (0.00124) | | | |
| Log(1 + Prev 1 Month CAR) | | | | -0.124*** | -0.205*** | -0.152*** |
| | | | | (0.00579) | (0.0108) | (0.0110) |
| Log(1 + Prev 1 Month CAR) × Log($Price_{t-2}$) | | | | 0.0284*** | | 0.0238*** |
| | | | | (0.00187) | | (0.00226) |
| Log(1 + Prev 1 Month CAR) × Log($Size_{t-2}$) | | | | | 0.0156*** | 0.00409*** |
| | | | | | (0.00114) | (0.00136) |
| Fama-MacBeth | Yes | Yes | Yes | Yes | Yes | Yes |
| Avg R-squared | 0.073 | 0.069 | 0.084 | 0.048 | 0.039 | 0.056 |
| Observations | 1,717,530 | 1,717,530 | 1,717,530 | 3,256,589 | 3,256,589 | 3,256,589 |
| Time Periods | 1,014 | 1,014 | 1,014 | 1,084 | 1,084 | 1,084 |

- Past returns negatively predict future returns
- The strength of this reversal varies more by price than by size

# Volume after splits




- Holding a stock's market cap constant, increased speculation should lead to higher volume
- Instead, volume drops after splits, consistent with some investors trading fixed numbers of shares

- Volume increases following reverse splits, consistent with some investors trading fixed numbers of shares